

COMMENT ON VIKOR METHOD BY COMBINING THIS METHOD AND LINEAR ASSIGNMENT

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Abstract

Purpose: the purpose of this comment is to propose a new algorithm which shows the same result of Linear Assignment and Vikor method.

Findings: For finding new methods to solve multi criteria problems, we need to know the older ones, so at the first of paper we present a comprehensive description of the steps that we need to solve problems by Vikor and Linear Assignment Method and after, we show three example that we solved with new method. At the end we will Compare the results of new method and the old one in Conclusion part.

Limitation: These two methods come from two different sub-model , vikor is belong to compromising subgroup and Linear assignment is belong to concordance subgroup.

Value : The new method does not need any condition that is use in last step of vikor method . One other advantage that this new way present is ranking all alternatives at the end.

Key words: Multiple criteria decision making (MCDM); Vikor method; Linear assignment Method; Dual condition

Paper type: comment paper

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Introduction

As regards the Dual condition that is use in Vikor method at The final section , we find that this method make user Confused , so we suggest to use R , S and Q (we will describe these signs in next parts) as equiponderant Criteria in assignment method for ranking the alternatives . In this paper we would like to present two example that the first one is simple and the second one is more complicated with more criteria.

We follow the first three steps of Vikor method and make a matrix with the help of R, S and Q and then solve this matrix by Linear assignment Method.

Literature Review

Decision making describes the process through which, the solution of certain problems can be chosen (Hwang and Yoon, 1995). Most important decisions in organizations are made by groups of managers or experts. Managers spend much of their time in decision related meetings (Huber, 1984). Balancing tradeoffs between objectives is even more important in groups than for individuals, because conflicting objectives and opposing viewpoints are inevitably going to exist. Sycara (1991) presented a framework for problem restructuring based on the goals and goal relationships of the negotiating parties, recognizing this multiplication of goals.

Decision making groups can range from cooperative with very similar goals and outlooks, to antagonistic with diametrically opposed objectives. Even in cooperative groups, conflict can arise during the decision process (Poole et al., 1991). If group members have different viewpoints, some method of aggregating preferences and reconciling differences are needed. MCDM methods have been developed to solve conflicting preferences among criteria for single decision makers (Corner and Kirkwood, 1991; Keeney and Raiffa, 1976; Korhonen et al., 1984; Saaty, 1980).

The concordance method generates a preference ranking which best satisfies a given concordance measure. The Linear Assignment Method is one of the examples in this family. In this method it is believed that an alternative having many highly ranked attributes should be ranked high [Hwang and Yoon, 1981].

The principle of MADM is to select the best alternative from several mutually exclusive alternatives based on their general performance of multiple attributes (or criteria). According to the types and the characteristics of decision problems, different MADM methods have been

designed and implemented in various domains such as Linear Assignment method, TOPSIS, ELECTRE, etc.

Among these approaches, linear assignment method [3] uses concordance concept and linear programming (LP) model to determine the rank of alternatives. Since this method only requests the ordinal data instead of cardinal data, it is easy to understand and to be implemented in different domains. However, this approach still has several weaknesses that can be improved. First of all, this approach can only deal with precise information. In reality, that the precise information is available is unrealistic. Secondly, this approach can only deal with two types of attributes: benefit type and cost type. For some problems, the best attribute value is not the highest value or the lowest value, but the target value. The importance of target type's attribute should not be ignored. Thirdly, this approach does not consider actual cardinal difference between alternatives on each attribute. Hence, even two alternatives have the same rank on two attributes the actual cardinal difference between alternatives on each attribute can be quite large.

In this method, each alternative priority at every attribute is used to achieve a Zero-One Programming model and through the solution of this model, all alternatives will be ranked. The main procedure of the linear assignment method for the selection of the best alternative from among those available is described by Asgharpour (1992), Azar (2002) and Hwang and Yoon (1981).

The Linear Assignment Method

The proposed linear assignment method to determine the proper ranking order of alternatives by applying linear assignment principle. The detailed algorithm of our approach is explained as follows:

Step1. For example we have a matrix like this:

$$D = \begin{matrix} & x_1 & \dots & x_n \\ \begin{matrix} A_1 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & \dots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mn} \end{bmatrix} \end{matrix}$$

Step2. we can determine the ranking orders of alternatives for each attribute. Determine the ranking order of alternatives for each attribute. Its attribute wise ranking matrix will become.

$$\begin{array}{c}
 x_1 \quad \dots \quad x_n \\
 r = \begin{array}{l}
 1st \\
 2nd \\
 3rd
 \end{array} \begin{bmatrix}
 A_i & \dots & A_i \\
 \vdots & \ddots & \vdots \\
 A_m & \dots & A_m
 \end{bmatrix}
 \end{array}$$

Step3. Assign the proper weight for each rank of individual alternative.

The traditional linear assignment model neglects the importance of actual cardinal difference between alternatives on each attribute. The proposed approach will consider actual cardinal difference and compute the weight for each rank of individual alternative.

$$\begin{array}{c}
 x_1 \quad \dots \quad x_n \\
 r = \begin{array}{l}
 1st \\
 2nd \\
 3rd
 \end{array} \begin{bmatrix}
 W(A_i) & \dots & W(A_i) \\
 \vdots & \ddots & \vdots \\
 W(A_m) & \dots & W(A_m)
 \end{bmatrix}
 \end{array}$$

Step4. Convert the weight matrix into concordance matrix. Based on the weight matrix, we can construct a concordance (square) matrix r, whose element r_{ij} is the summation of the weights for all attributes where alternative i is ranked j. For instance, the concordance matrix of above example will be computed as below:

$$\begin{array}{c}
 \begin{array}{ccc}
 1st & 2nd & 3rd
 \end{array} \\
 r = \begin{array}{l}
 A_1 \\
 \vdots \\
 A_m
 \end{array} \begin{bmatrix}
 W(A_i) & \dots & W(A_i) \\
 \vdots & \ddots & \vdots \\
 W(A_m) & \dots & W(A_m)
 \end{bmatrix}
 \end{array}$$

Step 5. Form the linear assignment (LP) model:

$$\max: \sum_{i=1}^m \sum_{k=1}^m r_{ij} \cdot P_{ik}$$

$$st: \sum_{k=1}^m P_{ik} = 1 ; i = 1, \dots, m$$

$$\sum_{i=1}^m P_{ik} = 1 ; k = 1, \dots, m$$

$$P_{ik} = 0 \text{ or } 1$$

Where $P_{ik} = 1$ if A_i is assigned to overall rank j . Otherwise, $P_{ik} = 0$

Step6. Solve the LP model. The result is:

$$P^* = \begin{bmatrix} 0 \text{ or } 1 & \dots & 0 \text{ or } 1 \\ \vdots & \ddots & \vdots \\ 0 \text{ or } 1 & \dots & 0 \text{ or } 1 \end{bmatrix}$$

Step7. The optimization options is:

$$A^* = A.P^* = [A_1 \quad \dots \quad A_m] \begin{bmatrix} 0 \text{ or } 1 & \dots & 0 \text{ or } 1 \\ \vdots & \ddots & \vdots \\ 0 \text{ or } 1 & \dots & 0 \text{ or } 1 \end{bmatrix}$$

The Vikor Method

Multi Criteria Decision Making (MCDM) methods is a branch of a general class of Operations Research models which deal with the process of making decisions in the presence of multiple objectives. This class is further divided into Multi Objective Decision Making (MODM) and Multi Attribute Decision Making (MADM) (Pohekarand Ramachandran , 2004). These methodologies share the common characteristics of conflict among criteria, incommensurable units, and difficulties in design/selection of alternatives (Huang et al., 1995).

The VIKOR method is an effective tool in MCDM. This method introduces the multicriteria ranking index based on the particular measure of “closeness” to the “ideal” solution, F^* (Opricovic and Tzeng, 2004). The compromise solution F^c is a feasible solution that is the “closest” to the ideal solution, and compromise means an agreement established by mutual concessions.

Within the VIKOR method, the various j alternatives are denoted as a_1, a_2, \dots, a_j . For alternative a_j the rating of the i_{th} aspect is denoted by f_{ij} , i.e., f_{ij} is the value of the i_{th} criterion function for the alternative a_j ; n is the number of criteria. The compromise ranking algorithm VIKOR has the following steps (Opricovic and Tzeng, 2004):

Step1. Determine the best f_i^* and the worst f_i^- values of all criterion functions, $i = 1, 2, \dots, n$. If the i_{th} function represents a benefit then $f_i^* = \max_j f_{ij}$ and $f_i^- = \min_j f_{ij}$, while if the i_{th} function represents a cost $f_i^* = \min_j f_{ij}$ and $f_i^- = \max_j f_{ij}$. so we will make :

$$f_i^* = \{f_1^*, f_2^*, \dots, f_n^*\}$$

$$f_i^- = \{f_1^-, f_2^-, \dots, f_n^-\}$$

Step2. Compute the values S_j and R_j , $J=1, 2, \dots, j$ by the relations :

$$S_j = \sum_{i=1}^n w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-)$$

$$R_j = \max_i [w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-)]$$

wherein W_i are the weights of criteria, expressing the decision maker's preference as the relative importance of the criteria. The weights of relative importance of the attributes may be assigned using AHP (Saaty, 2000). The steps are explained below in the following way:

1. Find out the relative importance of different attributes with respect to the objective. To do so, one has to construct a pair-wise comparison matrix using a scale of relative importance. The judgments are entered using the fundamental scale of the AHP. An attribute compared with itself is always assigned the value 1 so the main diagonal entries of the pair-wise comparison matrix are all 1. The numbers 3, 5, 7, and 9 correspond to the verbal judgments “moderate importance”, “strong importance”, “very strong importance”, and “absolute importance” (with 2, 4, 6, and 8 for compromise between the previous

values). Assuming n attributes, the pair-wise comparison of attribute i with attribute j yields a square matrix $A_{n \times n}$ where a_{ij} denotes the comparative importance of attribute i with respect to attribute j . In the matrix, $a_{ij} = 1$ when $i=j$ and $a_{ij} = 1/a_{ji}$

2. We need to know the vector $W = \{w_1, w_2, \dots, w_n\}$ which indicates the weight that each criteria is given in pair-wise comparison matrix A . To recover the vector W from A we outline a method in a two-step procedure:
 - For each of the A 's column divide each entry in column i of A by the sum of the entries in column i . This yields a new matrix, called A_{norm} (for normalized) in which the sum of the entries in each column is 1.
 - Estimate W_i as the average of the entries in row i of A_{norm} .

Once we have the Pair wise Comparisons matrix it is necessary checking it for consistency.

After Compute the values of S_j, R_j we have to go next step

Step3. Compute the values Q_j , by the relation:

$$Q_j = v(S_j - S^*) / (S^- - S^*) + (1 - v)(R_j - R^*) / (R^- - R^*)$$

Wherein

$$S^* = \min_j S_j ; S^- = \max_j S_j ; R^* = \min_j R_j ; R^- = \max_j R_j$$

And v is introduced as a weight for the strategy of maximum group utility, whereas $(1 - v)$ is the weight of the individual regret. The solution obtained by $\min_j S_j$ is with a maximum group utility ("majority" rule), and the solution obtained by $\min_j R_j$ is with a minimum individual regret of the "opponent". Normally, the value of v is taken as 0.5. However, v can take any value from 0 to 1.

Step4. Rank the alternatives, sorting by the values S , R , and Q in decreasing order. The results are three ranking lists. Propose as a compromise solution the alternative $A^{(1)}$ which is the best ranked by the measure Q (minimum), if the following two conditions are satisfied:

- a. Acceptable advantage . $Q(A^{(2)}) - Q(A^{(1)}) \geq DQ$ where $DQ = 1/(J - 1)$

and $A^{(2)}$ is the alternative with second position in the ranking list by Q ;

- b. Acceptable stability in decision making. The alternative $A(1)$ must also be the best ranked by S or/and R . This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility (when $v > 0.5$ is needed), or “by consensus” ($v \approx 0.5$), or with veto ($v < 0.5$).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed which consists of:

- c. Alternative $A^{(1)}$ and $A^{(2)}$ if only condition b is not satisfied, or
 d. Alternatives $A^{(1)}, A^{(2)}, \dots, A^{(m)}$ if the condition a is not satisfied. $A^{(m)}$ is determined by the relation $Q(A^{(m)}) - Q(A^{(1)}) < DQ$ for maximum n (the positions of these alternatives are “in closeness”).

Ranking the alternatives by the VIKOR method gives us, as a compromise solution.

Recommended model :

As mentioned vikor method has four step , in our solution The first three steps are repeated but then we will make a matrix with three Criteria that are x_R, x_S, x_Q and n rank per n alternatives like this : (M_n)

$$\begin{matrix}
 & x_S & x_R & x_Q \\
 \begin{matrix} rank1 \\ \vdots \\ rankn \end{matrix} & \begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix}
 \end{matrix} \quad \text{Figure (1)}$$

Now it is time to use assignment method and keep solving problem with this method.

Note1 : the new alternatives do not have weight , so we can consider same weight like this ($x_S, x_R, x_Q = 1$) or in more complicated situation that we have two or more alternative with same position it’s better to use these weight ($x_S: 1, x_R: 1, x_Q: 2$)

Numerical example:

Example1:

In order to apply the Vikor method, we have selected five alternatives and five Criteria. This problem is based on Metal selected projects and we just want to solve it with our way.

Material and their attributes

Attributes	304	321	316	2205	254
Yield strength(ys)	230	263	301	621	725
Tensile strength(TS)	500	589	602	841	901
Hardness (HRB)	73	80	83	293	310
PRE	17.5	17	23.1	35	43
Risk of failure	5	5	5	2	1
Cost(C) \$/Tm	3200	3800	5000	20000	40000

Step1.

	YS	TS	H	Cost	PRE	RF
	max	min	min	max	max	min
f_i^*	621	500	73	40000	43	1
f_i^-	230	841	293	3200	17	5

Step2.

	A1	A2	A3	A4	A5
S_j	0.8294	0.7646	0.7093	0.1400	0.6088
R_j	0.4495	0.4116	0.3679	0.0774	0.3656

Step3.

	A1	A2	A3	A4	A5
Q_j	1	0.920	0.803	0	0.727

Step4. In this step we have two approaches:

1. Continue the main way like the role that we said, 2. Use this new method, According to new method, this step is related to make matrix that we mentioned in figure (1).

$$M_n = \begin{matrix} & x_S & x_R & x_Q \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{matrix} & \begin{bmatrix} 0.8294 & 0.4495 & 1 \\ 0.7646 & 0.4116 & 0.92 \\ 0.7093 & 0.3679 & 0.803 \\ 0.1400 & 0.0774 & 0 \\ 0.6088 & 0.3656 & 0.727 \end{bmatrix} \end{matrix}$$

From this point onwards we are going to use assignment method Note2: pay attention that all of this three alternative should be considered negative. As result we will have:

Step 5.

$$\begin{matrix} x_S & x_R & x_Q \\ \begin{matrix} R1 \\ R2 \\ R3 \\ R4 \\ R5 \end{matrix} & \begin{bmatrix} A_4 & A_4 & A_4 \\ A_5 & A_5 & A_5 \\ A_3 & A_3 & A_3 \\ A_2 & A_2 & A_2 \\ A_1 & A_1 & A_1 \end{bmatrix} \end{matrix}$$

	Rank1	Rank2	Rank3	Rank4	Rank5
A ₁	0	0	0	0	3
A ₂	0	0	0	3	0
A ₃	0	0	3	0	0
A ₄	3	0	0	0	0
A ₅	0	3	0	0	0

If we accept that $w = \{1, 1, \text{ and } 1\}$ we will have:

Step6. $[A_1 A_2 A_3 A_4 A_5]. \begin{vmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{vmatrix} \rightarrow A_4 > A_5 > A_3 > A_2 > A_1$

If we do Vikor method exactly like main way, we will reach to same result. Ranking the alternatives by the Vikor method gives us, as a compromise solution, the alternative A_4 . This alternative, is the best ranked by Q. In addition, conditions 6-a and 6-b are satisfied as this alternative is also the best ranked by S and R.

But sometimes problems are more complicated than this one and we can't find best alternative simply. In this situation we have to use weights for each criterion. For making weighted matrix we will use entropy method.

Example 2:

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	C ₉	C ₁₀	C ₁₁	C ₁₂	C ₁₃	C ₁₄	C ₁₅	C ₁₆
A ₁	66	79	63	75	70	78	70	78	58	82	56	83	74	59	76	66
A ₂	70	72	72	68	69	72	74	72	67	68	57	75	72	66	77	66
A ₃	79	72	72	75	75	72	77	73	72	70	67	75	75	67	77	69
A ₄	77	70	74	71	77	74	79	80	75	67	63	70	74	70	80	65

After doing the calculations

rank \ index	x_S	x_R	x_Q
A ₁	0.595	0.095	0.752
A ₂	0.756	0.074	0.642
A ₃	0.366	0.065	0.008
A ₄	0.360	0.098	0.500

By using entropy method

$W = \{0.06, 0.1, 0.84\}$

Ranking:

rank \ index	x_S	x_R	x_Q
A ₁	A ₄	A ₃	A ₃
A ₂	A ₃	A ₂	A ₄
A ₃	A ₁	A ₁	A ₂
A ₄	A ₂	A ₄	A ₁

Number of repeat:

	Rank1	Rank2	Rank3	Rank4
A ₁	0	0	2	1
A ₂	0	1	1	1
A ₃	2	1	0	0
A ₄	1	1	0	1

	Rank1	Rank2	Rank3	Rank4
A ₁	0	0	0.1+0.06	0.84
A ₂	0	0.1	0.84	0.06
A ₃	0.1+0.84	0.06	0	0
A ₄	0.06	0.84	0	0.1

$$A^* = [A_1 \ A_2 \ A_3 \ A_4] \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = A_3 > A_4 > A_2 > A_1$$

In this example if we don't use weights, we can't rank alternatives.

Example 3:

$$x_S \quad x_R \quad x_Q$$

$$\begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{matrix} \begin{bmatrix} 1.47 & 0.54 & 0 \\ 1.49 & 0.8 & 0.86 \\ 2.53 & 0.92 & 1.37 \\ 2.06 & 0.80 & 1.13 \\ 1.56 & 0.72 & 0.87 \end{bmatrix}$$

By using entropy method

$$W = \{0.25, 0.08, 0.64\}$$

Ranking:

rank \ index		x_S	x_R	x_Q
	A ₁	A ₁	A ₁	A ₁
	A ₂	A ₂	A ₅	A ₂
	A ₃	A ₅	A ₄	A ₅
	A ₄	A ₄	A ₂	A ₄
	A ₅	A ₃	A ₃	A ₃

Number of repeat:

	Rank1	Rank2	Rank3	Rank4	Rank 5
A ₁	3	0	0	0	0
A ₂	0	2	0	1	0
A ₃	0	0	0	0	3
A ₄	0	0	1	2	0
A ₅	0	1	2	0	0

	Rank1	Rank2	Rank3	Rank4	Rank 5
A ₁	0.25+0.08+0.66	0	0	0	0
A ₂	0	0.25+0.66	0	0.08	0
A ₃	0	0	0	0	0.25+0.08+0.66
A ₄	0	0	0.08	0.25+0.66	0
A ₅	0	0.08	0.25+0.66	0	0

$$A^* = [A_1 \ A_2 \ A_3 \ A_4 \ A_5] \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} = A_1 > A_2 > A_5 > A_4 > A_3$$

As it is seen the alternative A₁ is the best rank by Vikor method and Linear Assignment method.

Conclusion:

If we consider to the two note that we have expressed, we will have new way with the Algorithm that we have expressed on Sixth step. We could solve 20 examples by this method and at the end we had similar results with previous methods.

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